Simplified Analytical Formulas for Thermal Blooming

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Propagation of a laser beam through a gas results in absorption of energy from the beam that raises the temperature of the gas and therefore lowers the index of refraction. The light rays are bent into regions of higher index away from the beam axis. This defocusing effect is referred to as thermal blooming. The theory of cw thermal blooming has been examined by several groups.[1] The theory is nonlinear and requires computer calculations.

Thermal blooming implies the existence of an optimum laser power. At low powers the peak focal plane irradiance increases linearly with increasing power, but for sufficiently high powers the focal spot is bloomed to so great an extent that the irradiance decreases with further increase in power. The laser power giving maximum peak focal plane irradiance is referred to as the optimum laser power, P_{op}.

A large number of computer runs have been carried out using an NRL propagation code for an infinite Gaussian beam. The data base thus generated was then used to develop a scaling law for peak irradiance, I_{pk}, of the form

$$I_{pk} = [I_{vac} \exp(-\alpha F)]/(1 + AP + BP^2), \tag{1}$$

where P is the laser power.[2] A and B are complicated functions of the laser and absorbing gas parameters. The present report gives simple formulas for P_{op} and the associated I_{pk} value, I_{op}. These formulas are fits, good to within 1–5% of values generated from the more general scaling law subject to the power optimization condition $\partial I_{pk}/\partial P = 0$. These fits were obtained by a three-variable separable-function approximation.

The parameters that influence thermal blooming are the absorption coefficient of the gas α, the component of wind across the beam v_o, the focal length F, the aperture size of the light source D, the laser power P, the beam sluing Ω, which is defined as positive when sluing into the wind, and the wavelength λN, where both optical quality and truncation or occlusion are accounted for in the theory by means of the factor N, called the number of times diffraction limited. $N = 1$ refers to a nontruncated Gaussian beam with a $1/e^2$ power point of diameter D. By employing length and time scale changes, these seven physical parameters can be reduced to five dimensionless parameters. For the range of parameter space specified below, it is found that the calculations for I_{pk} can be described using only the four dimensionless parameters

$$\begin{aligned} \xi &= \alpha F \\ \eta &= N\lambda F/D^2 \\ \zeta &= \Omega F/v_o \\ \beta &= \sqrt{2}\pi^2(3N_{mr}/C_s^2)(\gamma - 1)\alpha PD/\lambda^2 N^2 v_o, \end{aligned} \tag{2}$$

where N_{mr}, C_s, and γ are the molar refractivity of the gas, the speed of sound in the gas, and the ratio of specific heats, respectively.

The vacuum value for focal plane irradiance is given by

$$I_{vac} = P \exp(-\alpha F)\pi/2(\lambda F/D^2). \tag{3}$$

The thermally bloomed focal plane irradiance can be expressed as

$$\begin{aligned} I_{rel} &= I_{pk}/I_{vac} \\ &= I_{rel}(\beta, \xi, \eta, \zeta). \end{aligned} \tag{4}$$

A final, dimensionless parameter can be eliminated by consider. . $P = P_{op}$. At P_{op}, Eq. (4) takes the form

$$I_{rel} = I_{relop}(\beta_{op}, \xi, \eta, \zeta), \tag{5}$$

where

$$\beta_{op} = \beta_{op}(\xi, \eta, \zeta), \tag{6}$$

so

$$I_{relop} = I_{relop}(\xi, \eta, \zeta). \tag{7}$$

From Eqs. (2) and (6) it follows that

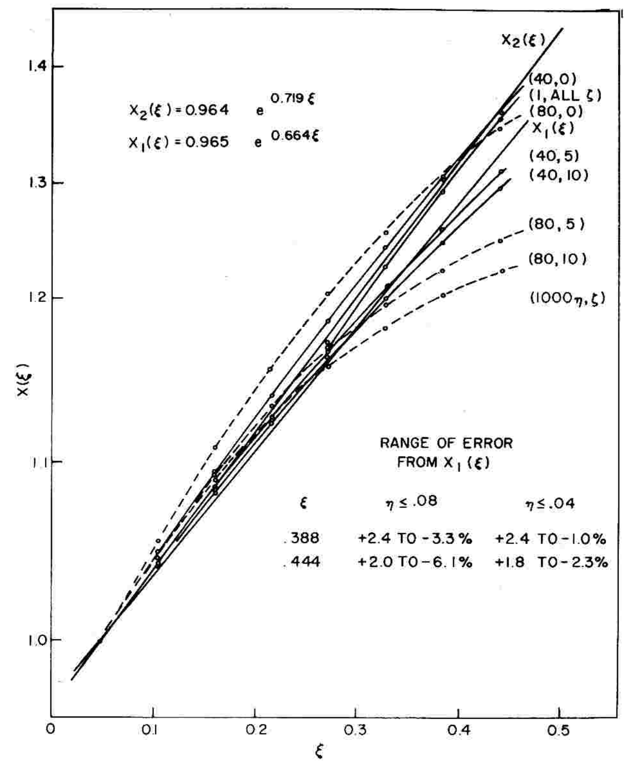

Fig. 1. Plots of $X(\xi)$. $X_1(\xi)$ is the preferred curve fit.

$$P_{op} = (N^2\lambda^2 v_o/\alpha D)\beta_{op}'(\xi, \eta, \zeta), \tag{8}$$

where β_{op}' is proportional to β_{op}. Equations (2), (3), (4), and (7) then give

$$I_{op} = \alpha v_o D\xi^{-2}\exp\cdot(-\xi)F(\xi, \eta, \zeta), \tag{9}$$

where $F(\xi,\eta,\zeta)$ is proportional to the product of $\beta_{op}'(\xi,\eta,\zeta)$ and $I_{relop}(\xi,\eta,\zeta)$. It is the function $F(\xi,\eta,\zeta)$, and with less accuracy $\beta_{op}'(\xi,\eta,\zeta)$, that can be fit over a chosen range of ξ, η, and ζ by a separable function approximation.

In the separable function approximation, $F(\xi,\eta,\zeta)$ can be written as

$$F(\xi, \eta, \zeta) = F(\xi_o, \eta_o, \zeta_o)X(\xi)Y(\eta)Z(\zeta), \tag{10}$$

where ξ_o, η_o, and ζ_o are fixed values chosen somewhere in the desired ranges of ξ, η, and ζ. The parameter ranges investigated are

$$\begin{aligned} 0.02 \le \xi \le 0.5 \\ 0.002 \le \eta \le 0.08 \\ 0 \le \zeta \le 10. \end{aligned} \tag{11}$$

Plots of the three product functions are given in Figs. 1, 2, and 3. From the fits to $X(\xi)$, $Y(\eta)$, and $Z(\zeta)$, it follows that

$$F(\xi, \eta, \zeta) \propto \exp(2\xi/3)\eta^{-3/4}(1 + 0.54\zeta), \tag{12}$$

and hence that

$$I_{op} \propto [v_o D^{2.5} \exp(-\alpha F/3)(1 + 0.54\Omega F/v_o)]/(\alpha N^{0.75}\lambda^{0.75}F^{2.75}). \tag{13}$$

The largest errors of ±6% relative to the general scaling law come from the fit of $Z(\zeta)$. Indications are that for high

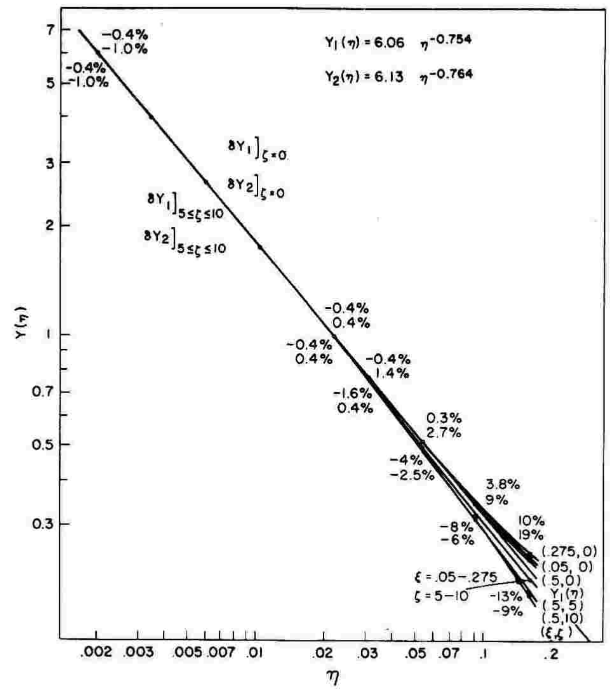

Fig. 2. Plots of $Y(\eta)$. The chosen curve fit is a slight clockwise rotation of $Y_1(\eta)$.

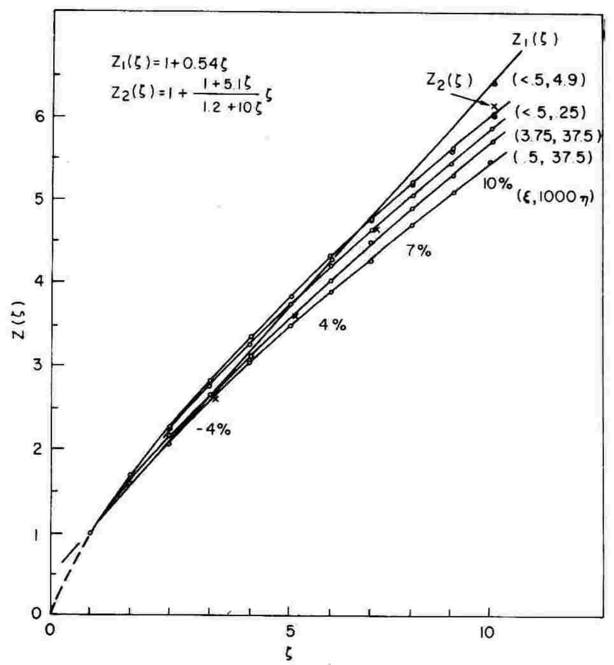

Fig. 3. Plots of $Z(\zeta)$. A simple straight line fit is used to approximate $Z(\zeta)$ for $\zeta \le 10$.

sluing with $\zeta \gg 10$, $F(\xi,\eta,\zeta)$ can no longer be approximated by a separable function. The accuracy of the scaling law is usually ±1% but occasionally reaches ±10% for some cases of high sluing.

The same approach has been applied to the function β_{op}', but in this case the results are more involved and the errors worse. The inherent error in defining P_{op} is probably at least ±15%. The best fits to the curve give

$$P_{op} \propto [v_0/(\alpha D)] \exp[2\xi/3\eta^{-\frac{3}{4}(1+0.007x)}](1 - 0.172x)$$
$$\times (\{\xi[(9.6 + 0.2\zeta)/(14 + \zeta)] - 0.0022\eta\} + 1) \pm 15\%, \quad (14)$$

where $x = \exp(-2\zeta/3)$. A simple, less accurate formula for P_{op} is

$$P_{op} \propto \{[v_o D^{0.5} \exp(-2\alpha F/3) N^{1.25} \lambda^{1.25} (1 + c\Omega F/v_o)]/$$
$$(\alpha F^{0.75})\} \pm 22\%, \quad (15)$$

where c ranges from 0.5 to 0.7, compared with the value of 0.54 in Eq. (12). I_{op} has the form

$$I_{op} \propto P_{op} \exp(-\alpha F)(D/N\lambda F)^2, \quad (16)$$

where P_{op} is given by Eq. (15). It is not surprising that the optimum laser power should increase with exit pupil diameter. $P_{op} \propto D^{0.5}$, and I_{op} varies as $D^{2.5}$ rather than as $D^{2.0}$. More generally, $P_{op} \propto v_o D^{0.5} N^{1.25} \lambda^{1.25} \exp(-2\alpha F/3)/\alpha F^{0.75}$; and hence $I_{op} \propto v_o D^{2.5} \exp(-\alpha F/3)/\alpha N^{0.75} \lambda^{0.75} F^{2.75}$. The dependence of I_{op} is not the usual low-power form. Both P_{op} and I_{op} are proportional to the heat-removing component of wind blowing across the beam. Similarly, the optimum usable power decreases with α. For small αF, the main dependence of I_{op} on α is an inverse rather than exponential proportion. I_{op} drops off more as I/F^3 and I/F^2 and depends only weakly on N, but strongly on D.

It was shown in Eqs. (13) and (14) or (15) that I_{op} and P_{op} can be expressed by analytical formulas accurate over a specified region of parameter space. A separability approximation was used to develop these simplified formulas, showing the dependence on each parameter separately.

References

1. The main publications of the five laboratories working on thermal blooming are at NRL, J. N. Hayes, P. B. Ulrich, and A. H. Aitken, Appl. Opt. **11**, 257 (1972); NRL Report 7413 (1971). At Avco Everett, J. Wallace, J. Opt. Soc. Am. **62**, 373 (1972); J. Wallace and M. Camac, J. Opt. Soc. Am. **60**, 1587 (1970). At Lincoln Labs., L. C. Bradley and J. H. Herrmann, J. Opt. Soc. Am. **61**, 668 (1971); Project Report LTP-10, 12 July 1971. At AFWL, internal reports by C. B. Hogge. At Aberdeen Proving Grounds, internal reports by H. Breaux.
2. P. B. Ulrich, J. N. Hayes, and A. H. Aitken, J. Opt. Soc. Am. **62**, 298 (1972).